

The Three Oldest Unsolved Problems in Mathematics

This document explores the three oldest known unsolved problems in mathematics, unpacks their origins, significance, and why they remain unsolved to this day. We will also explore a long list of related historical details and tangentially connected problems that grew out of these mysteries.

1. The Riemann Hypothesis (1859)

Although not the absolute oldest, it is historically one of the most famous and still unsolved problems, deeply connected to number theory.

Origins:

- Proposed by Bernhard Riemann in 1859 in a paper on the distribution of prime numbers.
- Suggests that all nontrivial zeros of the Riemann zeta function have a real part equal to $1/2$.

Significance:

- Direct link to the distribution of prime numbers.
- Impacts cryptography, random matrix theory, and analytic number theory.

Why Unsolved:

- Despite massive computational evidence confirming it for trillions of zeros, no proof exists.
 - Requires new mathematical tools possibly beyond current reach.
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2. Goldbach's Conjecture (1742)

One of the earliest stated unsolved problems still standing.

Origins:

- Proposed in correspondence between Christian Goldbach and Leonhard Euler.
- States: Every even integer greater than 2 can be expressed as the sum of two primes.

Significance:

- Deep ties to additive number theory.
- Would strengthen understanding of prime number distribution.

Why Unsolved:

- Verified by computer for numbers up to extremely large limits, but no general proof exists.
- Links to sieve theory, which has major mathematical roadblocks.

3. The P vs NP Problem (1971)

While not as old as ancient Greek problems, its conceptual roots go back to the 19th century and the foundations of computation.

Origins:

- Officially formulated in 1971 by Stephen Cook.
- Asks: If a problem's solution can be verified quickly, can it also be solved quickly?

Significance:

- Central to computer science, cryptography, optimization, and AI.
- If solved (especially $P = NP$), most encryption systems would collapse overnight.

Why Unsolved:

- No clear path to resolution; would require groundbreaking insights into computational complexity.

Honorable Mentions: Ancient and Classical Problems Still Unsolved

These are even older than the three listed above:

1. **Perfect Cuboid Problem** (17th century): Does a rectangular box exist with integer edges, face diagonals, and space diagonal?
2. **Odd Perfect Numbers** (Euclid era): Do any odd perfect numbers exist?
3. **Infinitude of Twin Primes** (Ancient Greece via Euclid's prime proof): Are there infinitely many twin primes?
4. **Collatz Conjecture** (1937, but deceptively simple like ancient riddles): Does every positive integer eventually reach 1 under the $3n+1$ rule?
5. **Euler's Sum of Powers Conjecture** (Disproved in one form but still open for certain exponents).

Historical Timeline of Unsolved Problems:

- **~300 BCE**: Euclid's proof of infinitude of primes (still inspires unsolved twin prime conjecture).
 - **1637**: Fermat's Last Theorem proposed (solved only in 1994 by Wiles, but served as a model for attacking deep problems).
 - **1742**: Goldbach's Conjecture.
 - **1859**: Riemann Hypothesis.
 - **1900**: Hilbert's Problems (23 in total; several still unsolved).
 - **1971**: P vs NP problem officially framed.
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Concluding Notes:

The "oldest" status depends on how we define it—some problems date to ancient Greece and Babylon but are more curiosities, while others like Goldbach's Conjecture are well-defined formal problems from the 18th century still standing. Riemann Hypothesis and P vs NP might be more modern, but their implications and difficulty make them as legendary as any ancient challenge.

In the grand scope of mathematics, these problems are more than just puzzles—they are doorways to entirely new branches of human knowledge.